

## The change of fractal dimensionality in the recovery and recrystallisation process

This article has been downloaded from IOPscience. Please scroll down to see the full text article.

1990 J. Phys.: Condens. Matter 2 3879

(<http://iopscience.iop.org/0953-8984/2/16/017>)

View [the table of contents for this issue](#), or go to the [journal homepage](#) for more

Download details:

IP Address: 171.66.16.103

The article was downloaded on 11/05/2010 at 05:53

Please note that [terms and conditions apply](#).

## LETTER TO THE EDITOR

# The change of fractal dimensionality in the recovery and recrystallisation process

Xiaowei Wang, Linke Dong and Liangyue Xiong

Institute of Metal Research and International Centre for Materials Physics, Academia Sinica, Shenyang, People's Republic of China

Received 8 November 1989, in final form 19 January 1990

**Abstract.** This letter explains the distinction between using the fractal dimensionality and the grain size in describing the change of grain shape when the concept of fractal dimensionality is introduced to the study of the recovery and recrystallisation process in materials. The fractal dimensionality of pure Zn was measured in the recovery and recrystallisation process. From these results a relationship between the fractal dimensionality and the energy and a physical interpretation of the dimensionality were deduced.

Recovery and recrystallisation in metals are among the most complicated phenomena in materials science. There are the obvious changes of structure, both for the sample as a whole and locally within the sample, and the physical properties and mechanisms relating to this process. In past studies, it has been very difficult to investigate the relation between the complex structural changes and the physical features in detail because of the lack of information obtained from direct observation.

The measurement of grain size is often the most direct and quantitative approach for observing the changes in the grain in materials. But a hypothesis must be included in making traditional measurements [1], i.e. the irregular spatial shape of the grain is approximated to be a simple and regular geometrical shape (e.g. spherical) and thus the grain size can be measured only approximately. The size measured obviously cannot be exact, and hence does not accurately reflect the changes in the grain characteristics. Some researchers have used the method of topology classification to obtain a grain size in each class [2, 3, 5]. However, real grain boundaries are not composed of straight lines: many boundaries are tortuous and show shape variations caused by the differing curvature radii. Therefore the real grain shape cannot be described by simple geometrical parameters because the grain shape is not uniform. If the formation of the grain shape is interpreted in terms of energy and mechanical balance, there ought to be some law governing the change of grain shape. This law would be the key to defining the relationship between the law that describes the irregular shape of the whole and the physical law that determines the internal changes occurring during recovery and recrystallisation.

In recent years, a new concept of fractal geometry has been advanced by the famous mathematician Professor B B Mandelbrot. This theory has opened up wide vistas for searching for the aforementioned law in many complex phenomena. Our studies are intended to introduce the new concept into the study of the recovery and recrystallisation

problem. We have started from a new viewpoint and try to use a new approach to study the recovery and recrystallisation process. From the viewpoint of fractal geometry [4], the grain network consists of many grains with various shapes and samples having the same physical features existing in the same self-similar configuration. The network has a different pattern after deformation and recrystallisation. The motion and formation of high-angle boundaries bring about network deformations and there are self-similarities in the distribution throughout the structure and in the changes of the grain shape. The change of grain shape always has the same uniform tendency as the recovery and recrystallisation process, e.g. polygonisation during the initial stages. Therefore the fractal dimensionality of the grain shape was measured at a different stage. Note that the dimensionality measured is that of the whole grain network not the change of each individual grain. The change of each grain will, though, affect the change of fractal dimensionality of the whole network. If the law of fractal dimensionality obtained arises from some law of natural physical change, it is obviously more appropriate to the actual circumstances that, when describing the relationship between the change of the structure and the physical nature, the grain size is replaced by the fractal dimensionality.

Some very interesting results were obtained from the experiment using a 99.99% Zn sample. Initially the sample was an ingot casting. The sample was cold rolled, annealed at 350 °C, deformed by 50% under hydrostatic pressure, annealed again and deformed again. At the end of this process the sample had deformed by 40.8%. A portion of the deformed sample, near its centre, (i.e. in a well distributed deformation zone) was cut into several tablets. Then these tablets were annealed at different temperatures all for the same length of time, i.e. 15 minutes. Three groups of samples were obtained, as follows:

*Group A:*

---

No	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
<i>T</i> (°C)	40	60	80	100	120	140	160	180	200	220	240	260	280	300	320	350

---

*Group B:* The treatment conditions were the same as for group A but the samples were cut from a different section.

*Group C:* A group of samples were taken again from No 2 to No 10.

Note that groups A and B were from the same sample but were cut from different cross sections. Group C was annealed in another furnace. Group A, B and C samples had the same deformation states.

After the heating treatment, the sample was prepared by the metallography technique. The grain boundary was corroded twice by a 6% HCl and alcohol solution and a 10:10:80 HF, distilled water and H<sub>2</sub>O<sub>2</sub> solution. Finally the grain boundary of the sample was clearly visible. The sample was enlarged 200 times under a NEOPHOT 21 microscope and the micrographs of the samples annealed at various temperatures were obtained. These photographs were then measured under the Joyel-Loebl auto-image analyser with the proportionality set to about 1:1. In other words, when the figures were measured the enlargement times were not the same for every group because of the distance of the focal length from the pick-up camera. The total area of each figure was about 13 × 11 cm<sup>2</sup>. The area and perimeter of each grain in the figure were measured

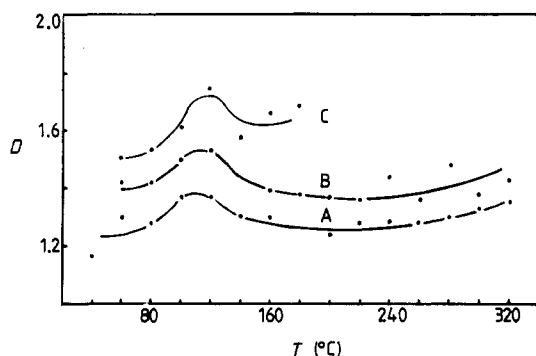


Figure 1. The change of fractal dimensionality versus the annealing temperature.

and the fractal dimensionality was calculated from the grain network at different temperature points to make use of the relationship of area to perimeter suggested by Professor B B Mandelbrot, i.e.

$$\ln P_i = C + (D/2) \ln A_i \quad (1)$$

where  $P_i$  and  $A_i$  are the perimeter and area of grain  $i$  respectively,  $C$  is the intersection distance of the regression straight line and  $D$  is the fractal dimensionality. The results are shown in figure 1 as a plot of  $D$  against temperature  $T$ . A very interesting law is demonstrated by this figure, i.e. every curve is basically the same but the absolute values of  $D$  at each of the temperature points are not the same. We attempted to analyse and clarify the results as follows.

(i) *Fractal dimensionality and energy change.* An obvious feature of figure 1 is that a peak always occurs in the initial stage with the change between the fractal dimensionality of the grain network and the annealing temperature, and that this peak temperature is near the recrystallisation temperature of the sample. The fractal dimensionality first increases, then decreases and then increases again. To explain this phenomenon, we considered two factors from the analysis of the energies.

We first looked at the deformation, or stored, energy. In general, deformed metals have more stored energy. The release of energy is the main driving force in the recovery and recrystallisation process, but in the recovery process there is no obvious energy change if the measurement of grain size is used, because the change during this stage is mainly in the micro-structure, i.e. in the polygons caused by the rearrangement of dislocations [6-9]. In particular, for metals having a greater stacking fault energy, the polygonisation is easier to achieve and the recrystallisation process is a little restrained [9]. There is competition on the release of energy. For the metals that recover easily the release of energy during the recovery stage is more than at any other stage. After the recrystallisation process starts the entire grain network will be changed by the formation of new nuclei from the elimination of the original grains. At this stage, not only are there changes of micro-structure but there are also changes in the distribution of the grains. The latter changes are dominant. The changes of fractal dimensionality are caused by the duplication of the two processes. In the recovery process the fractal dimensionality is increased by the larger release rate of stored energy. In the initial stage of recrystallisation, the formation of new nuclei also tends to increase the fractal dimensionality; then, with the homogenisation of the grain the fractal dimensionality will tend to decrease. The start of the decrease in the release rate of the energy is responsible for the

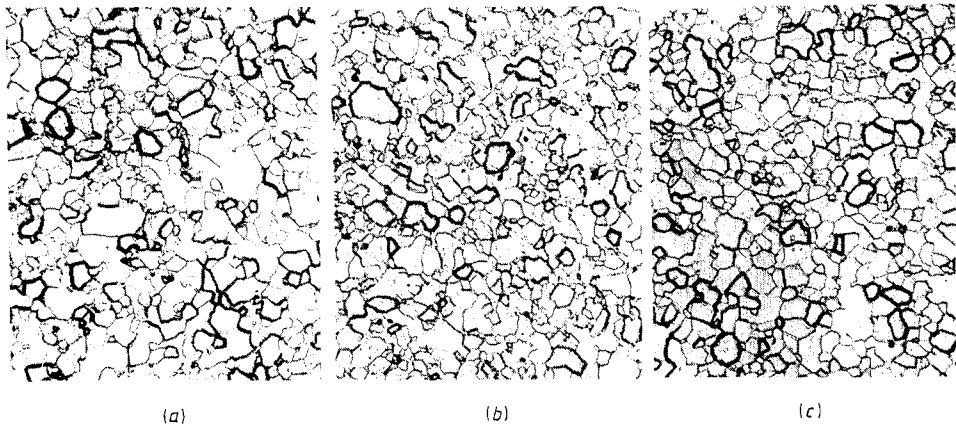
peak of the fractal dimensionality change. In other words, there is a relation between the change of the fractal dimensionality and the release rate of energy. In addition, the temperature of recrystallisation is seen to be related to the change in the release rate of energy.

Secondly, we analysed the surface energy. The release of deformation energy is mainly in the initial stages of recovery and recrystallisation. After the deformation energy is eliminated, the driving force for boundary migration can only come from the surface energy of the grain. In general, the grains that have less than six edges become contracted and the grains that have more than six edges are developed. Because the grain distribution is changed from inhomogeneous to homogeneous, the fractal dimensionality of the grain network will change from decreasing with the fractal dimensionality to approaching a constant. For the perfect crystal  $D = 1$ .

In the process of grain development, although the inhomogeneity of the distribution is decreasing, the state of actual thermodynamic equilibrium still cannot be reached because the grain boundary curve still changes with the change of the temperature and is always in a sub-stable state. Therefore, it is only when the samples are kept for a long time under certain temperature conditions and the boundary migration rate is constant that the fractal dimensionality will tend to a constant. For changing temperature, there will still be a stable change in the fractal dimensionality. If the secondary recrystallisation occurs, the fractal dimensionality will obviously change again. When the time spent at each temperature point is too short for the secondary recrystallisation to occur, this process is basically a homogeneous process. Only at 350 °C, near the melting point, does the fractal dimensionality tend to increase.

(ii) *Fractal dimensionality and grain size.* The crystallisation process is a structure-sensitive process caused by the non-equilibrium character of the system. The grain size (i.e., in general, the grain diameter) can only describe the average scale change of the grain—large or small. A considerable approximation is made regarding the effect of the shape and distribution of the grain. In real materials, it is very difficult to achieve a wholly homogeneous distribution of the grains and it is impossible for all grains to have the same shape. Therefore, for these figures and distributions that have irregular surfaces and are produced in a random way but do in reality obey certain laws, it would be more exact and sensitive to describe relationships by making use of the fractal dimensionality, based on the self-similarity concept. An important distinction is that measurements are no longer for Euclidean geometry, but are now made for fractal geometry based on direct observation. In a normal recrystallisation figure, grain size always changes monotonically with the temperature. This cannot explain the fundamental reason for the change of grain size. The fractal dimensionality can connect the change of grain shape with the driving force, and the fractal dimensionality can also reflect the change in the fine structure. For example, images (a), (b) and (c) in figure 2 show respectively the grain changes before and after recrystallisation. These data show that the change of fractal dimensionality not only depends on the distribution of grains but also on the level of fractal structure. In particular, the change of boundary extent may be the direct reason for the change of the level of fractal structure. Certainly there is a feature common to the measurement of fractal dimensionality and grain size. All the measurements ignore the effect of boundary length, but fractal dimensionality only ignores the width of the boundary and it includes the effect of the boundary shape.

(iii) *The physical meaning of recrystallisation fractal dimensionality.* In the application of fractal dimensionality, the self-similarity is always defined within a certain range. This is different from the initial mathematical definition because any mathematical theorem must be adapted to suit practical problems. In general, the fractal



**Figure 2.** The grain images before and after recrystallisation. (a) 60 °C,  $D = 1.422$ . (b) 100 °C,  $D = 1.502$ . (c) 260 °C,  $D = 1.358$ .

dimensionality must be unique, but in this experiment, the absolute value of  $D$  appears to differ because of the difference in the enlargement times. In order to explain these phenomena, we need to relate fractal dimensionality to the physical features of this process.

From figure 1, although the values of  $D$  are different at every temperature point, the trends of the curves tend to be the same. It shows that there is a common change mechanism, except that the times to the change of fractal dimensionality are greater. The change in  $D$  caused by the increased times is mainly due to the choice of 'yardstick'. The choice of yardstick has a different meaning to the choice of scaling. The scaling is defined by the physical characteristics, but the yardstick may be arbitrarily changed. Therefore, the diversity of the values of  $D$  is only related to the measurement, and the type of change of  $D$  with temperature is related to the physical properties, i.e. energy  $U$ , entropy  $S$ , defect configuration  $r$ , boundary migration rate  $G$ , etc. The value of  $D$  is a function of temperature. The above analysis gives us

$$D = C + F(U, S, r, G, T, \dots). \quad (2)$$

From the generalised dimensionality concept [8], we may write

$$D_F = \inf \{D\} \quad (3)$$

where  $D$  is the measured dimensionality and  $D_F$  is the Hausdorff dimensionality.

Besides these physical characteristics, fractal dimensionality can help us reveal natural laws which cannot be discovered by any other approach. It also helps in our understanding of the internal structural mechanisms and other associated physical factors. The introduction of the new measurement concept might help us to find specific laws governing many complex phenomena.

The authors are very grateful to Dr A J Song and J Xu, who work in the Institute of Applied Ecology, Academia Sinica, for their help with the experiment. This work was supported by a fund from the International Centre for Materials Physics, Academia Sinica.

**References**

- [1] Qin G J 1987 *Quantitative Metallography* (Chengdu: Sichuan Science and Technology Press)
- [2] Kurtz S K and Carpay F M A 1980 *J. Appl. Phys.* **51** 5745–54
- [3] Kurtz S K 1980 *J. Appl. Phys.* **51** 573
- [4] Mandelbrot B B 1983 *The Fractal Geometry of Nature* (New York: Freeman)
- [5] Pande C S 1988 *Acta Metall.* **36** 2161–6
- [6] Cahn R W 1970 *Physical Metallurgy* (Amsterdam: North-Holland)
- [7] Ha K F 1983 *The Micro-Theory of Metal Mechanical Properties* (Beijing: Science Press)
- [8] Ren G W and Yang Z A 1989 *Fractal Theory and its Application* (Chengdu: Sichuan University Press) p 21
- [9] Gorelik S S 1981 *Recrystallization in Metals and Alloys* (Moscow: MIR)